Standard Error of the Mean and Confidence Intervals for the Mean

The precision of the mean of a sample of data, as an estimate of some unknown or "true" vlaue of the mean of the population, can be described using the *standard error of the mean*, or by describing or plotting *confidence intervals for the mean*.

The standard error of the mean

The standard error of the mean is given by

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}},$$

where σ is the standard deviation of the population and n is the sample size. In practice, because the standard deviation of the population is not known, the standard deviation of the sample, s, is used in place of σ. The standard error of the mean (which is sometimes referred to as the standard deviation of the mean), has the nice property of increasing as the variability of the data increases, and decreasing as the sample size increases.

Confidence interval for the mean

The confidence interval for the mean is the range of values that is likely to enclose the "true" value of the mean some particular proportion of the time (e.g. as in repeated sampling of the same processor or phenomena). The confidence interval for the mean can be described by noting that a) means are normally distributed (which is implied by the central limit theorem) and consequently b) the normal distribution can be used to describe the probability of observing different values of the mean. The $(1 = \alpha)\%$ confidence interval for the mean is written as

$$\overline{X} = \pm Z\sigma_{\overline{X}},$$

Where $\overline{X}$ is the sample mean, $\sigma_{\overline{X}}$ is the standard error of the sample mean, and Z is a value (obtained using the cumulative density function (cdf) of the normal distribution, such that $(\alpha / 2)\%$ of the area under the pdf lies to the left of $-Z$ (i.e. $(\alpha / 2)\%$ of the values of $\overline{X}$ observed in practice are less than $-Z$), and $(\alpha / 2)\%$ of the area lies to the right of $+Z$ (i.e. $(\alpha / 2)\%$ of the values of $\bar{X}$ observed in practice are less than $+Z$). The width of the confidence interval depends on the magnitude of the standard error, and the desired precision.