The Moran Statistic

The Moran statistic, or spatial autocorrelation coefficient, is given by

$$I = \left(\frac{n}{\sum_{i=i}^{n} \sum_{j=1}^{n} w_{ij}} \right) \left(\frac{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \left(x_i - \overline{x}\right)\left(x_j - \overline{x}\right)}{\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2} \right)$$

where

$n =$ the number of points or spatial units

$x =$ the variable of interest

$\overline{x} =$ the mean of x, and

$w_{ij} =$ the spatial weight describing the adjacency or distance between the i-th and j-th point.

The expected value of Moran's I that would be obtained when there is no spatial autocorrelation is $E(I) = 1/(n-1)$.

The significance of I can be judged by calculating the variance of I and then comparing the following statistic to the standard normal distribution

$$Z = \frac{I - E(I)}{\operatorname{var}(I)}.$$

Under the assumption that the observed pattern of points is just one out of many possible patterns composed of n points, the variance of I is given by

$$\operatorname{var}(I) = \frac{nS_4 - S_3 S_5}{(n-1)(n-2)(n-3)\left(\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \right)^2}$$

where

$$S_1 = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} (w_{ij} + w_{ji})^2\right) / 2$$

$$S_2 = \sum_{i=1}^{n} \left(\sum_{j=1}^{n} w_{ij} + \sum_{j=1}^{n} w_{ji} \right)^2$$

$$S_3 = \frac{n^{-1}\sum_{i=1}^{n}(x_i - \overline{x})^4}{(n^{-1}\sum_{i=1}^{n}(x_i - \overline{x})^2)^2}\ S_4 = (n^2 - 3n + 3)S_1 - nS_2 + 3(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij})^2 \text{, and}$$

$$S_5 = S_1 - 2nS_1 = 6\left(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\right)^2 .$$

In practice, values of $Z \geq 2.0$ or $Z \leq -2.0$ (p-values $\leq$ 0.05) indicate significant spatial autocorrelation.